LTI Systems: Cascades

ECE 3640 Discrete-Time Signals and Systems
Utah State University

Motivation

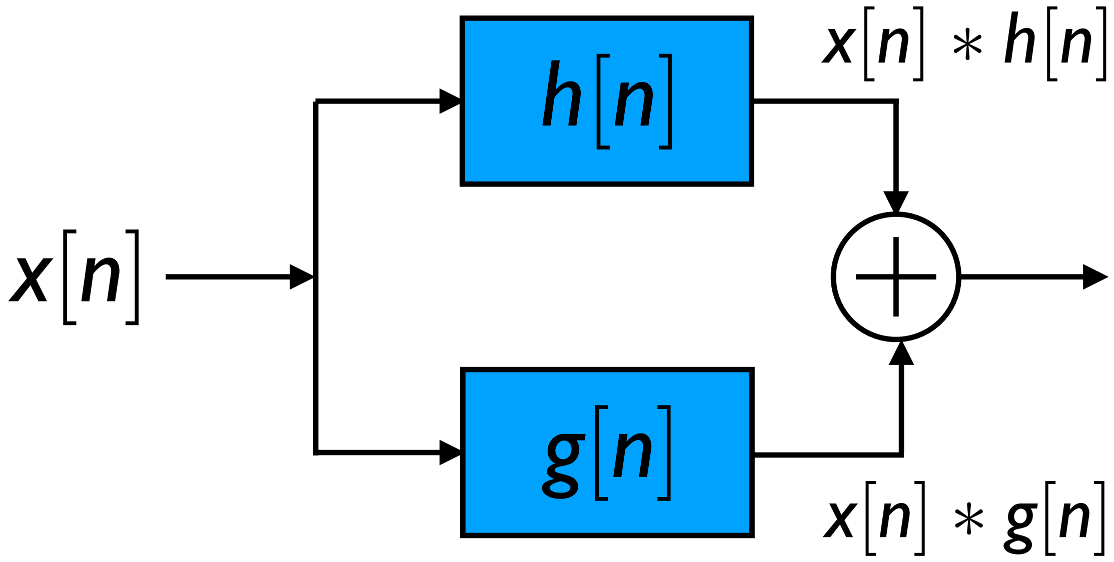
Individual LTI systems are interesting.

But LTI systems become really interesting when used to model physical systems and combined in interconnected cascades.

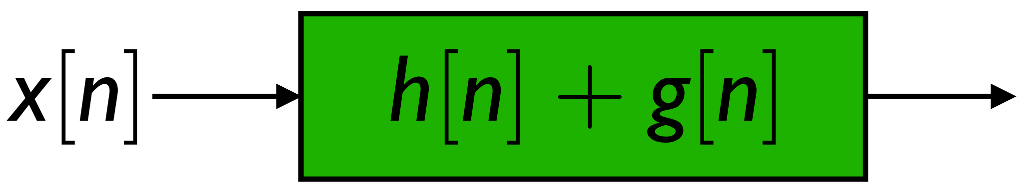
There are two kinds of interconnections:

- Parallel cascades
- Series cascades

Parallel LTI Cascade



$$
\begin{aligned}
y[n] &= x[n] * h[n] + x[n] * g[n] \\
&= \sum_k x[k]h[n-k] + \sum_k x[k]g[n-k] \\
&= \sum_k x[k]\left(h[n-k] + g[n-k]\right) \\
&= x[n] * (h[n] + g[n])
\end{aligned}
$$

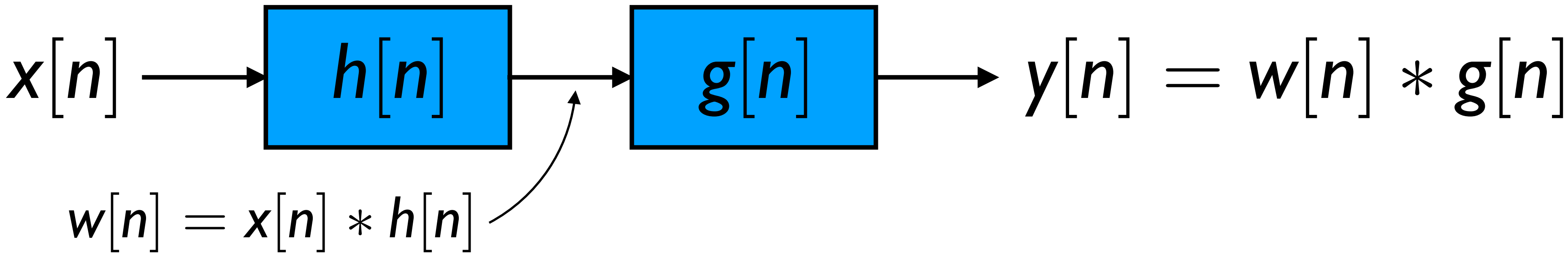


$$y[n] = x[n] * (h[n] + g[n])$$

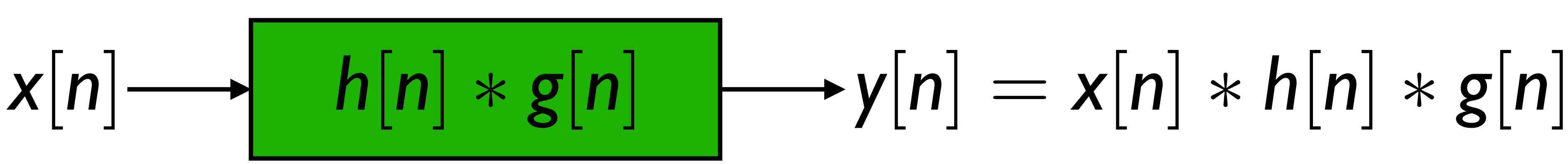
Convolution obeys distributive law:

$$x * h + x * g = x * (h + g)$$

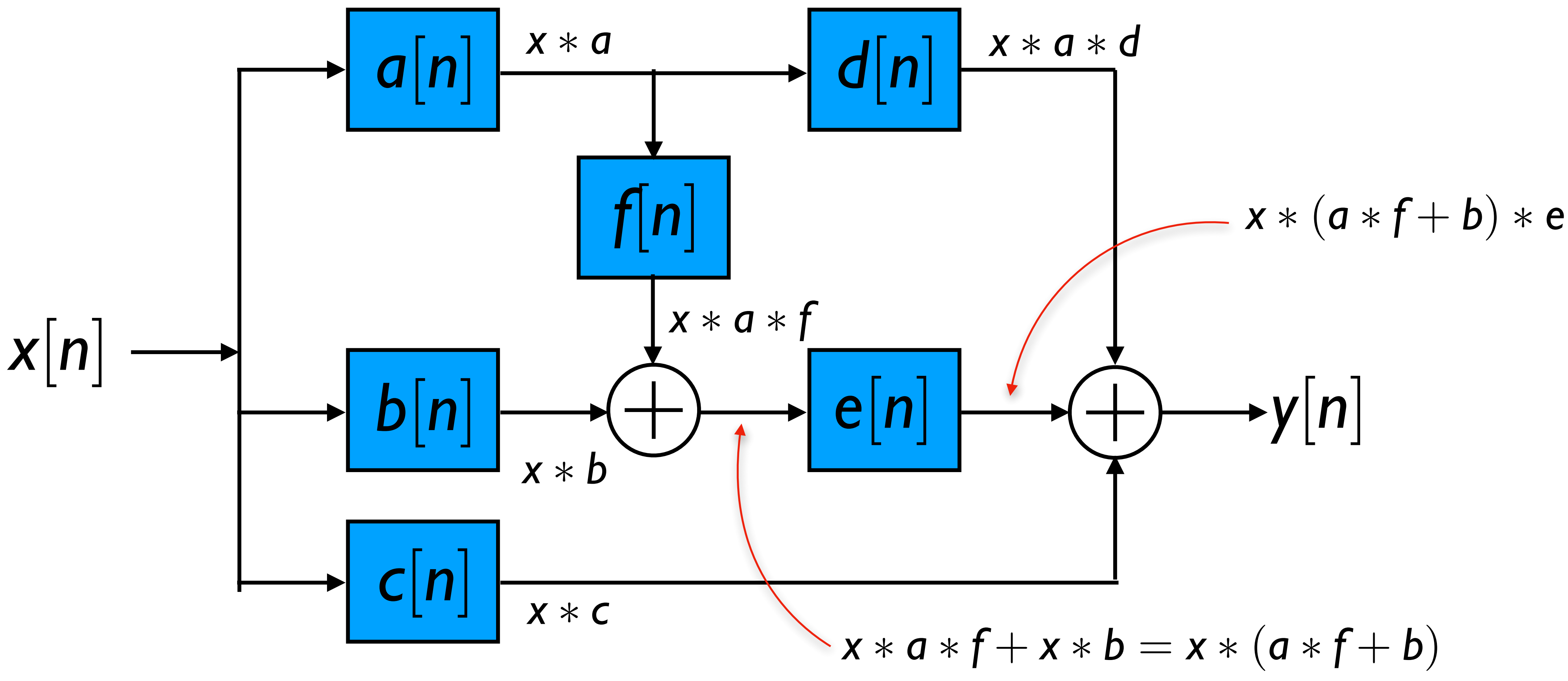
Series LTI Cascade

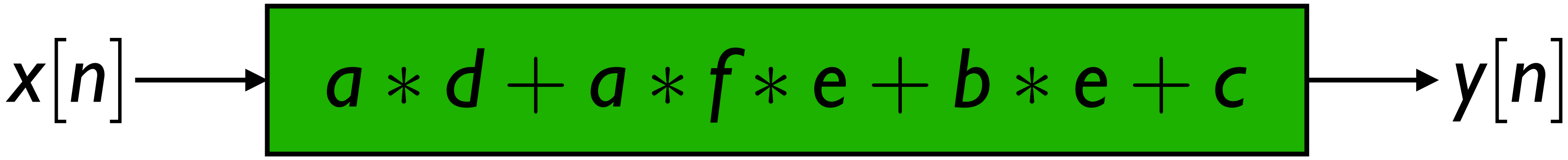


$$
\begin{aligned}
y[n] &= \sum_k w[k]g[n-k] \\
&= \sum_k \left(\sum_m x[m]h[k-m] \right) g[n-k] \\
&= \sum_m x[m] \sum_k h[k-m]g[n-k], \quad i = k-m,\ k = m+i \\
&= \sum_m x[m] \sum_i h[i]g[(n-m)-i] \\
&= \sum_m x[m]b[n-m] = x[n] * b[n] \\
b[n] &= \sum_i h[i]g[n-i] = h[n] * g[n]
\end{aligned}
$$



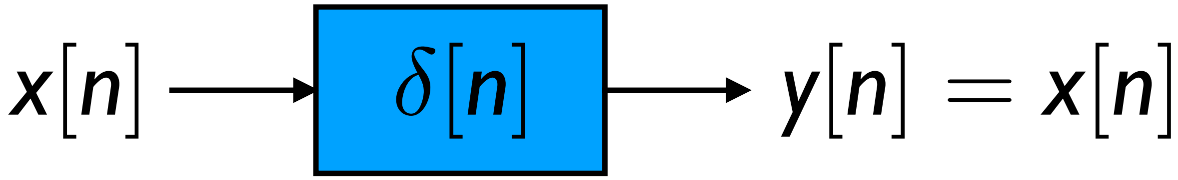
Complicated LTI Cascades





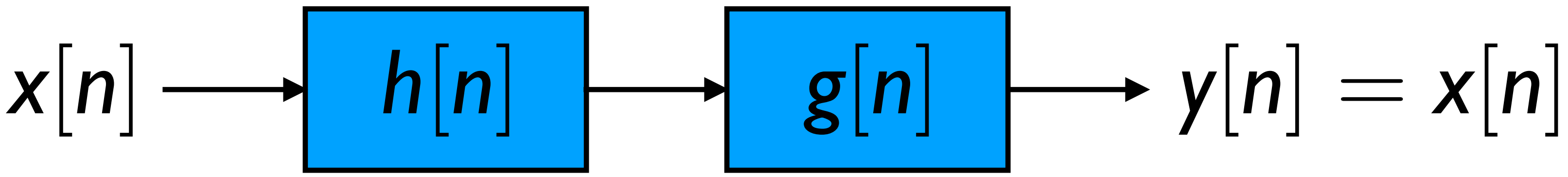
When adding impulse responses, line up the zero lag points.

LTI Identity and Inverse Systems



(nondistorting)

This is the identity system.



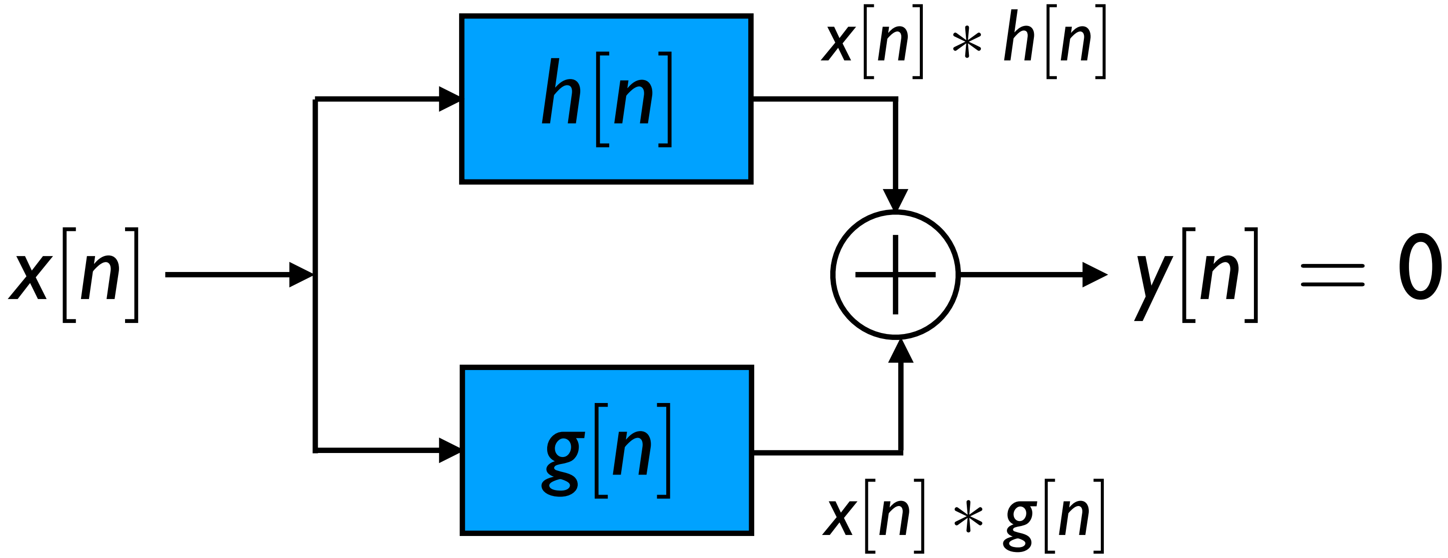
If $x[n] * h[n] * g[n] = x[n]$, then $h[n] * g[n] = \delta[n]$.

$g[n]$ is the inverse system of $h[n]$

Usually we are satisfied if $h[n] * g[n] = \delta[n-d]$.

(Overall distortion is only a delay.)

LTI Cancelation Systems



If $y[n] = x[n] * (h[n] + g[n]) = 0$, then $g[n] = -h[n]$.

The two paths have equal and opposite responses.

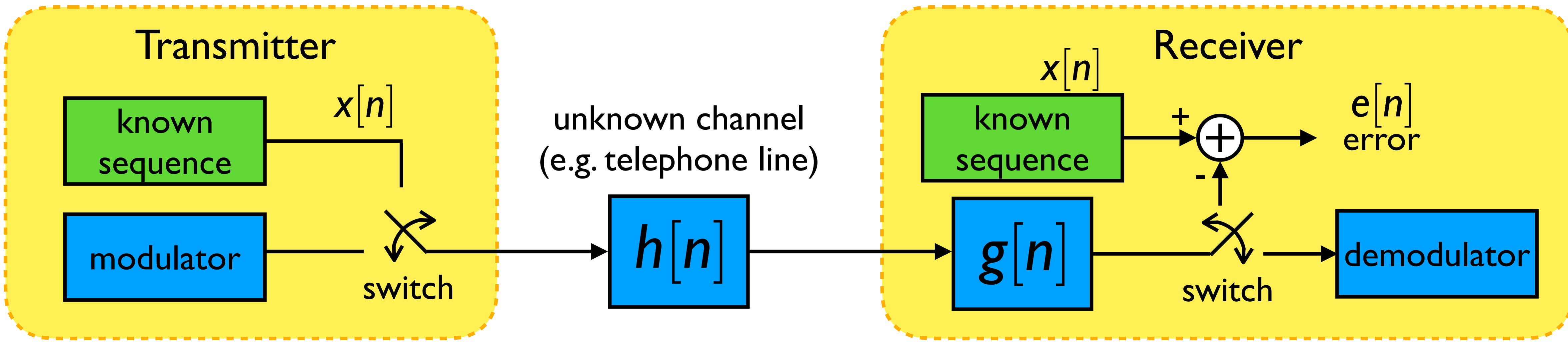
$g[n]$ cancels $h[n]$

Applications

Let’s apply these principles to understand real systems.

- Parallel cascades
- Series cascades
- Identity systems
- Inverse systems
- Cancellation systems

Equalization in Fax and Modems



Suppose the known sequence is switched in at transmitter and receiver, then

$$
\begin{aligned}
e[n] &= x[n] - x[n] * h[n] * g[n] \\
&= x[n] * \delta[n] - x[n] * h[n] * g[n] \\
&= x[n] * (\delta[n] - h[n] * g[n])
\end{aligned}
$$

The error is zero $e[n] = 0$ when $h[n] * g[n] = \delta[n]$.

$g[n]$ is the inverse of the unknown channel $h[n]$

$g[n]$ is called an equalizer

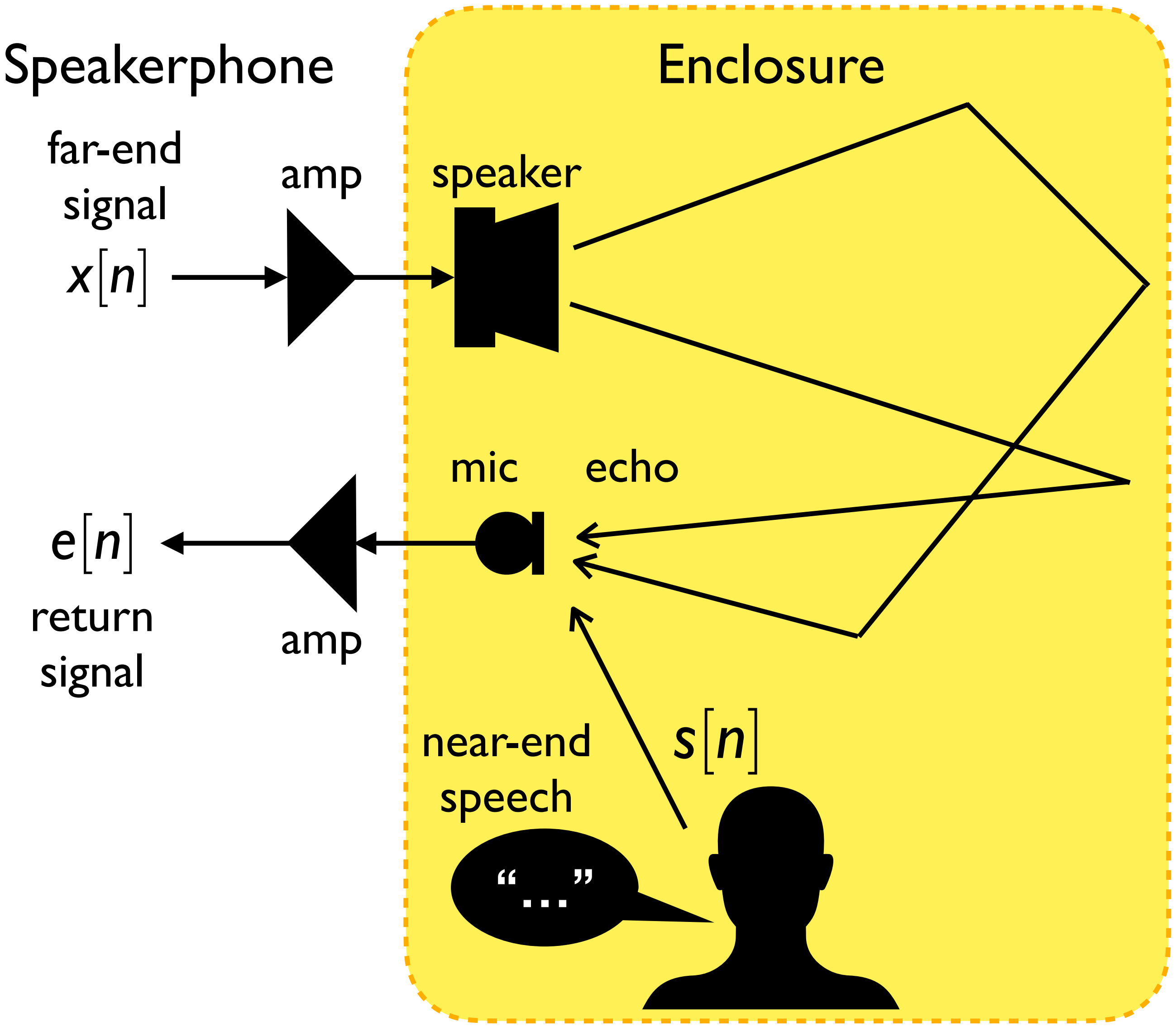
Acoustic Echo Cancellation

Speakerphone

Smart Speaker

Acoustic Echo Cancellation

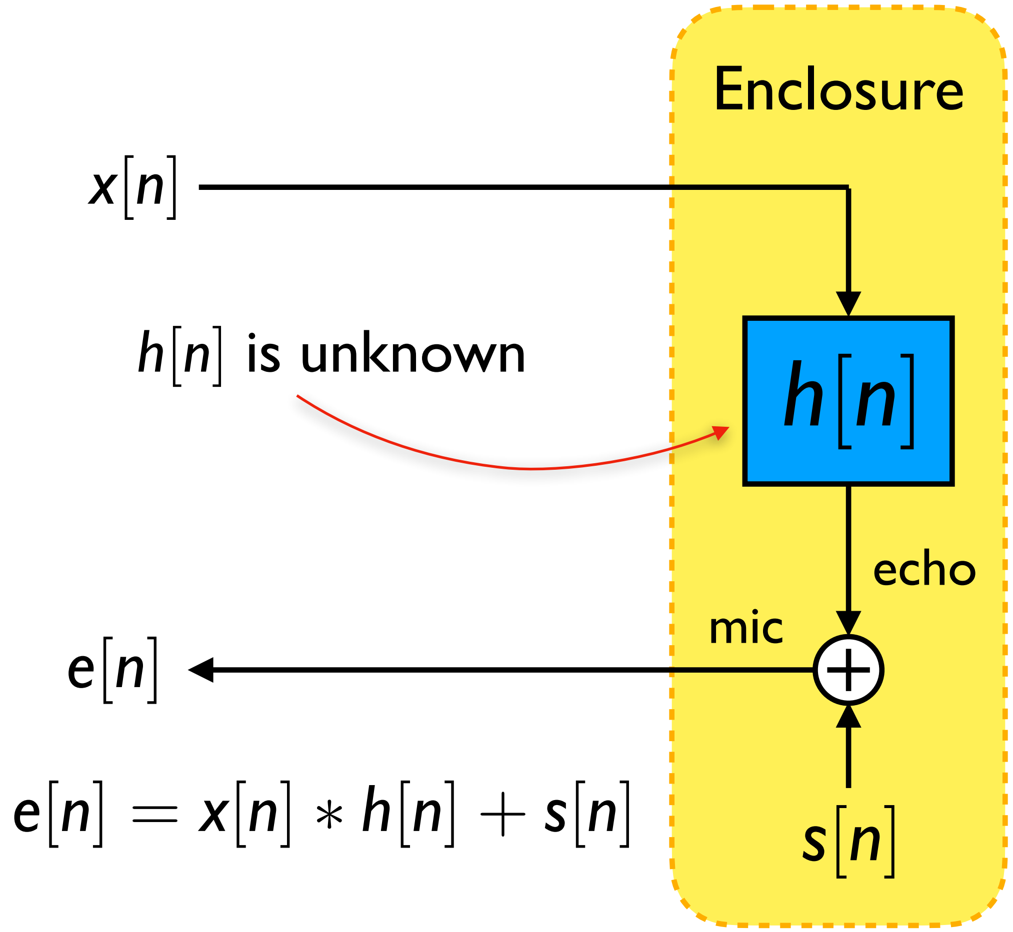
Physical System



Echo consists of superposition of delayed and scaled copies of far-end signal.

Need to remove/cancel the echo on the return signal.

LTI Model of Physical System

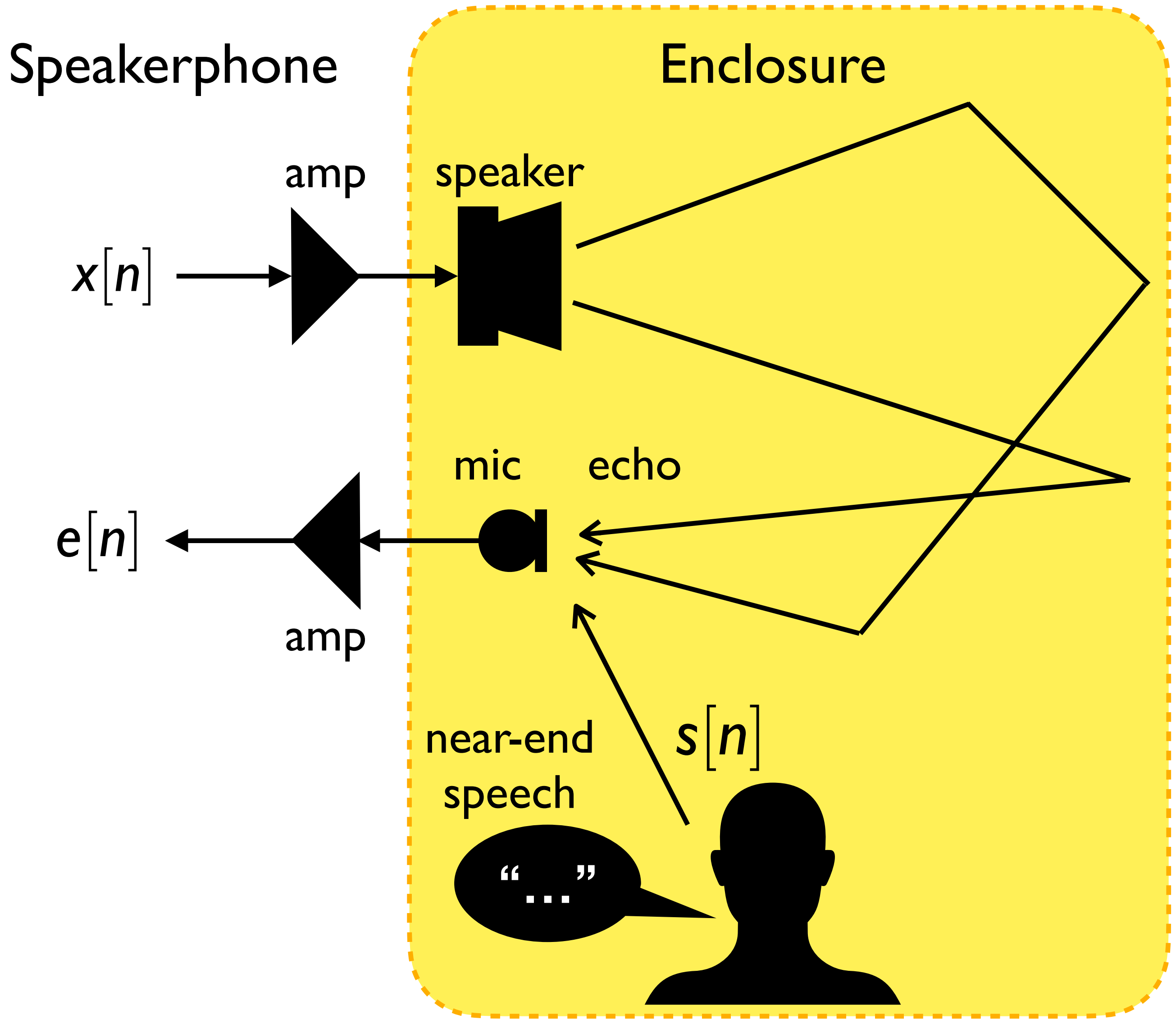


Use an LTI system to model the impulse response of the enclosure.

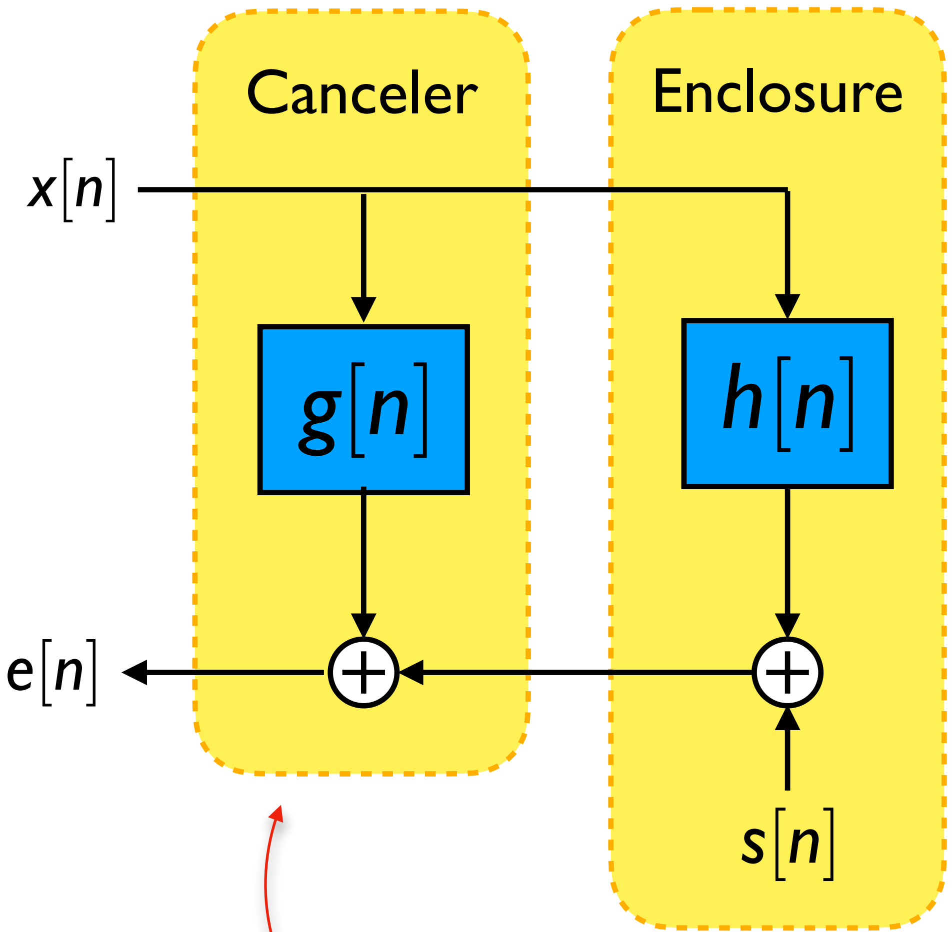
We want $e[n] = s[n]$.

Acoustic Echo Cancellation

Physical System



LTI Model of Physical System



Insert a canceler inside the speakerphone.

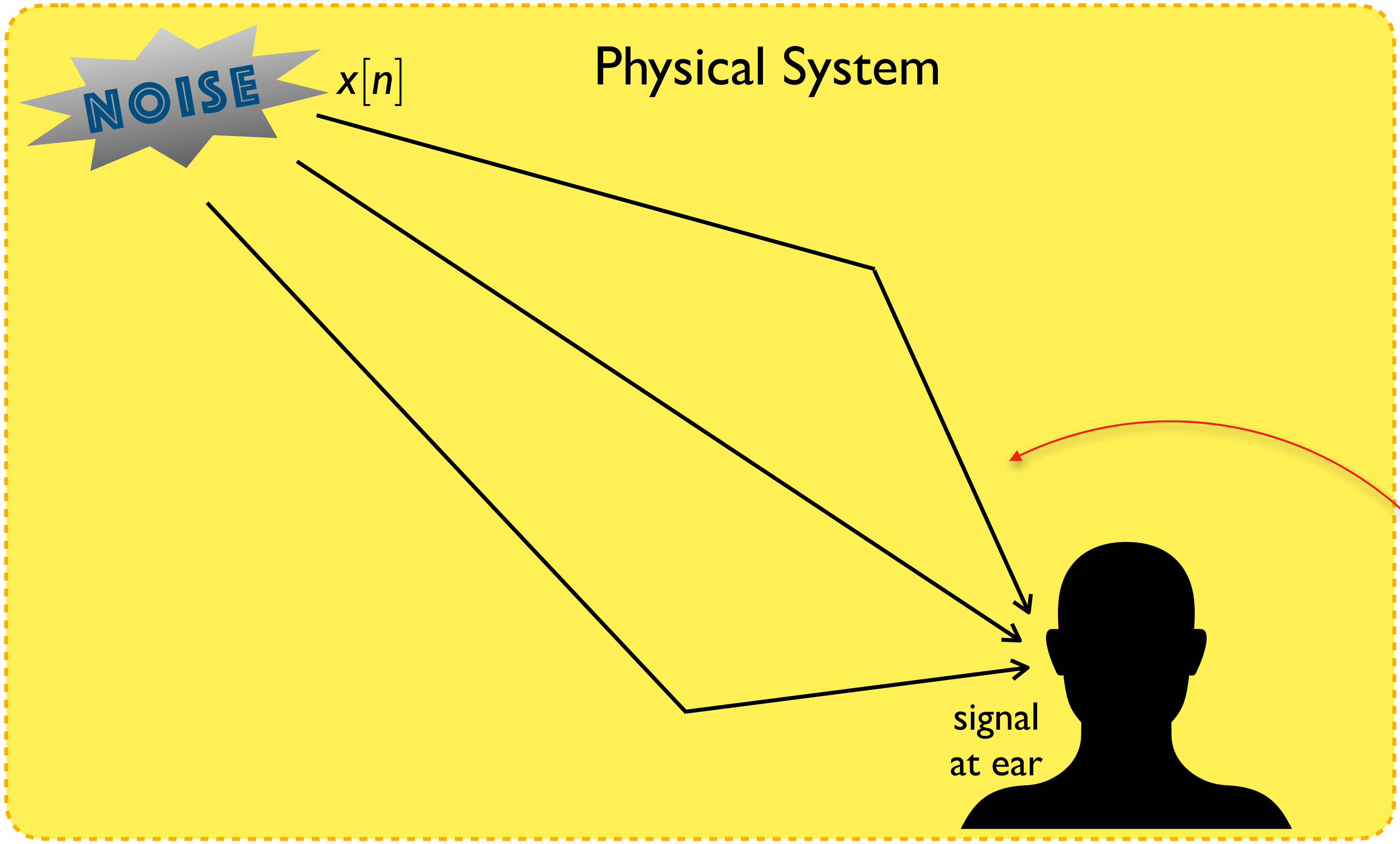
$$e[n] = x[n] * h[n] + x[n] * g[n] + s[n]$$
$$= x[n] * (h[n] + g[n])$$

Adapt $g[n]$ to make $e[n] = 0$, when $s[n] = 0$.

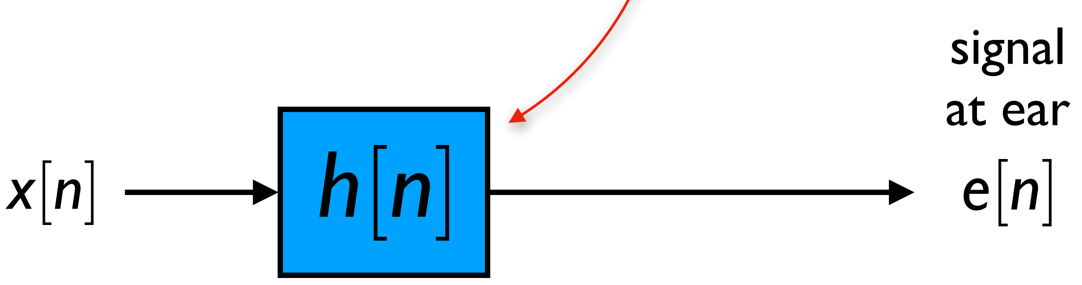
This is called system identification.

Noise Cancelling Headphones

Noise Cancellation



Response of acoustic path is unknown.

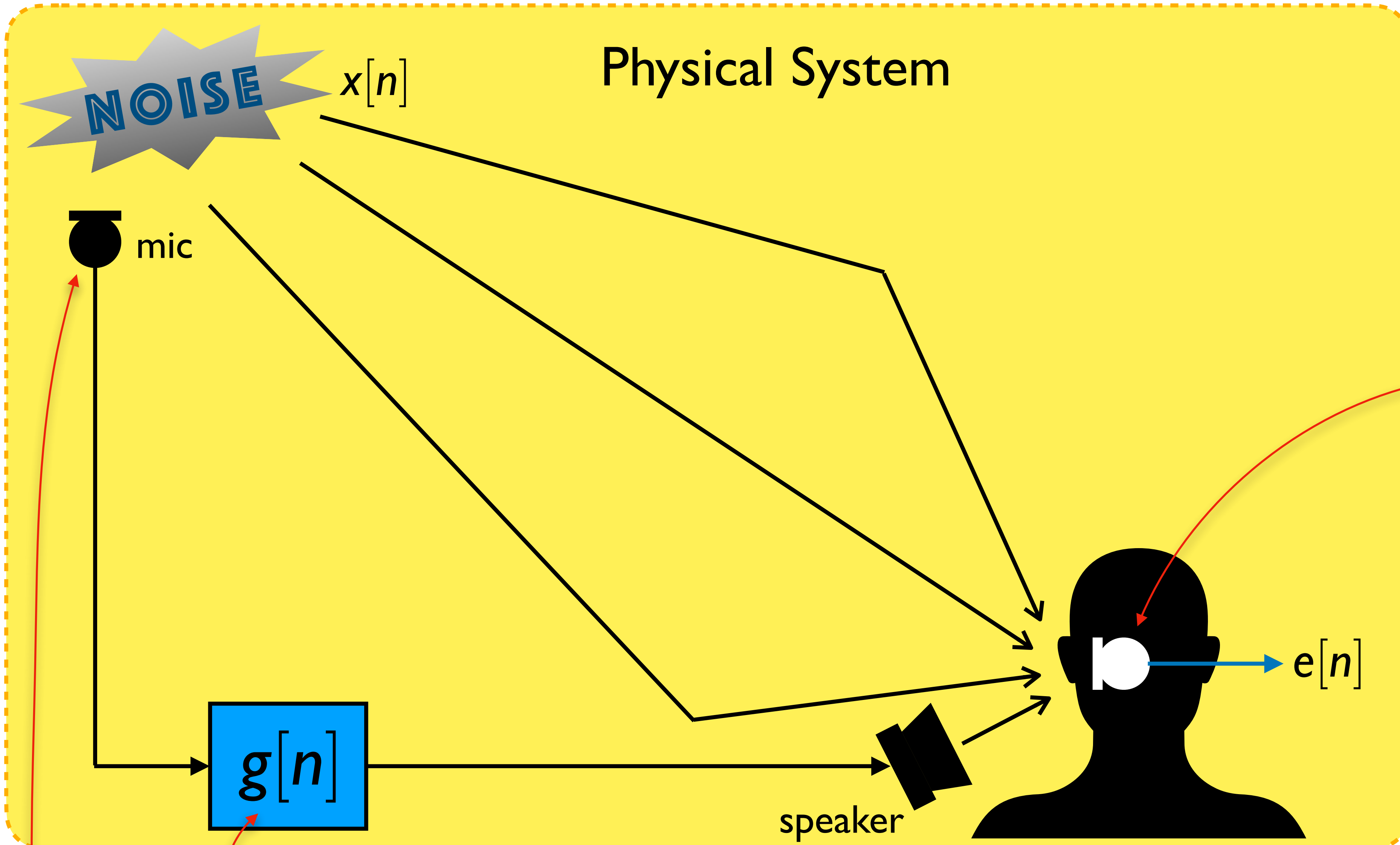


Person hears annoying noise.

Model acoustic path using LTI system.

Want to cancel noise. To cancel, create a parallel path …

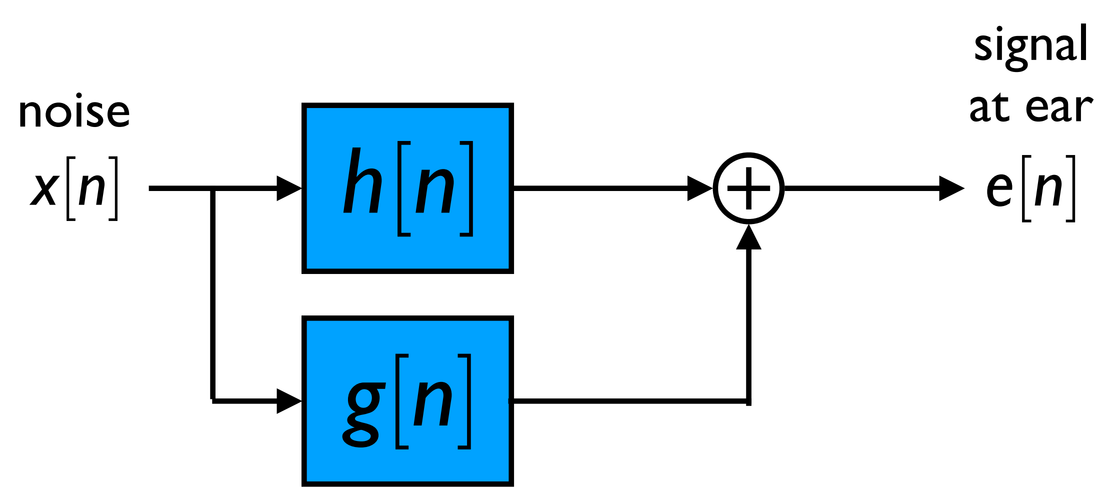
Noise Cancellation



Add a microphone near the ear to measure signal at ear.

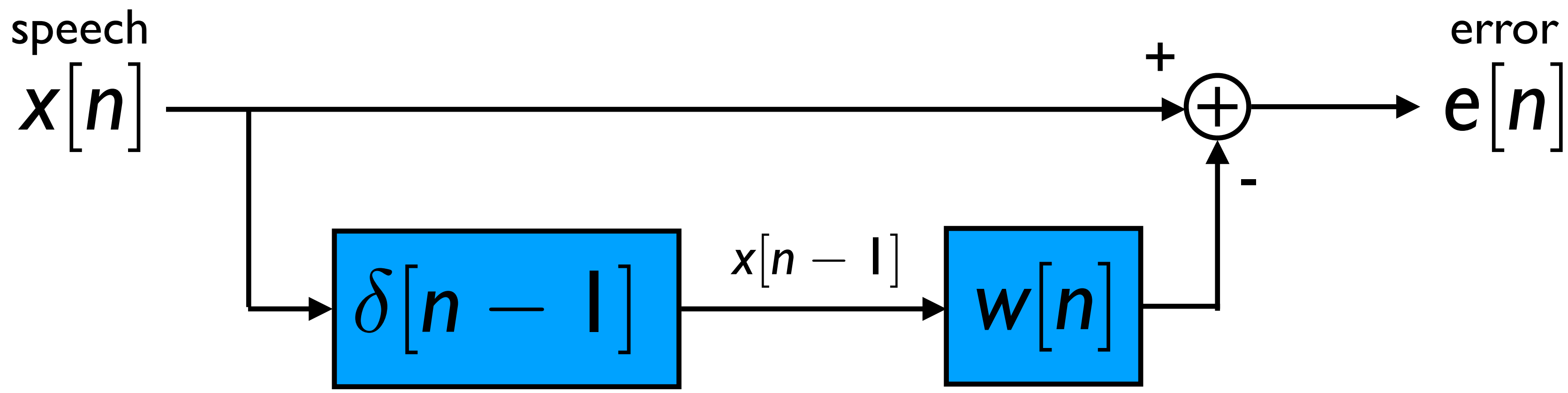
Add LTI system to act as noise canceler.

Add a microphone to measure the noise.



Adapt $g[n]$ to make $e[n] = 0$.

Linear Prediction



$$e[n] = x[n] - x[n] * \delta[n-1] * w[n]$$
$$= x[n] - x[n-1] * w[n]$$

Adapt $w[n]$ to make $e[n] = 0$.

$$x[n] = \sum_{k=0}^{N} x[n-1-k]w[k] + e[n]$$

Example: 20 ms of flute. *x[n]* (blue), prediction (red), *e[n]* (yellow)