ECE 3640 - Discrete-Time Signals and Systems

LTI Systems: IIR Computation

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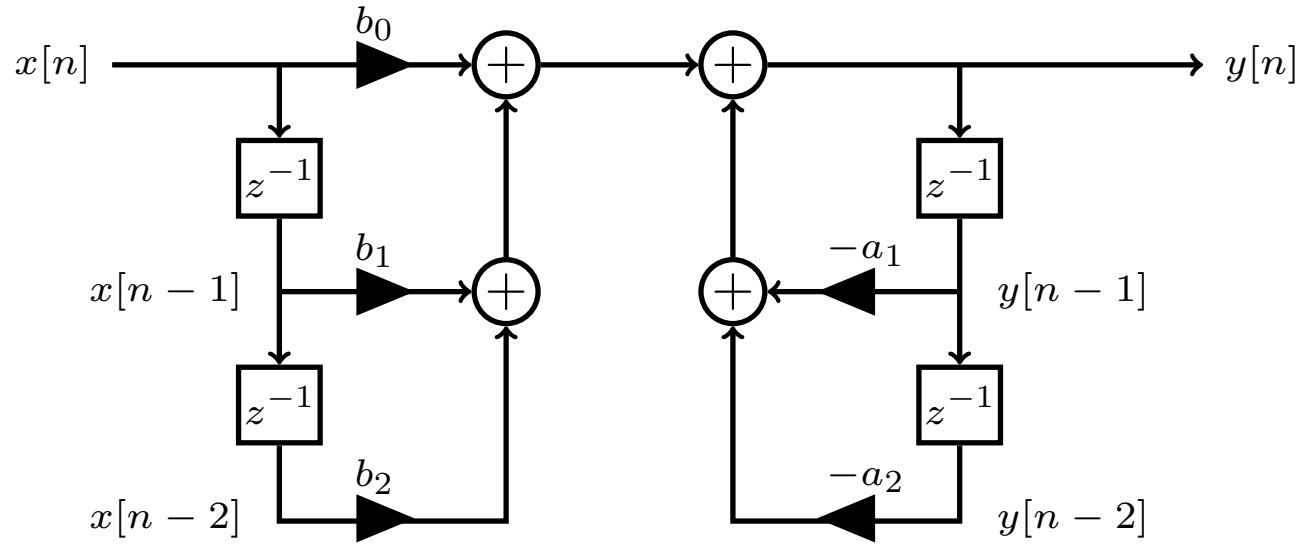
main points

- compute the output of DT IIR LTI systems (difference equation)
- compute the output of DT FIR LTI systems (filtering, convolution)

difference equation & block diagram



$$y[n] = \sum_{i=0}^{2} b_i x[n-i] - \sum_{i=1}^{2} a_i y[n-i]$$

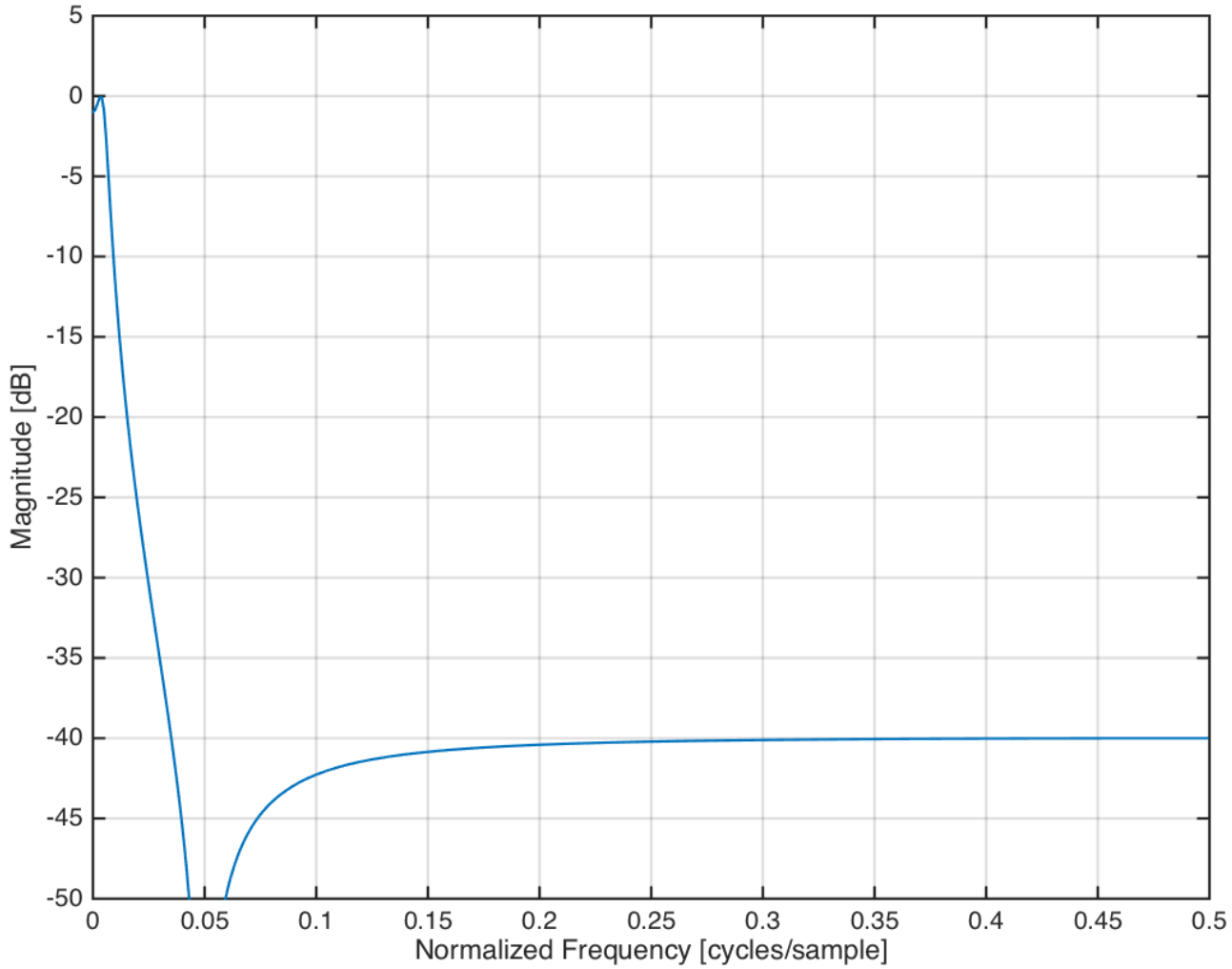
$$= b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] - a_1 y[n-1] - a_2 y[n-2]$$

$$b_0 = 0.0101, \quad b_1 = -0.0192, \quad b_2 = 0.0101$$

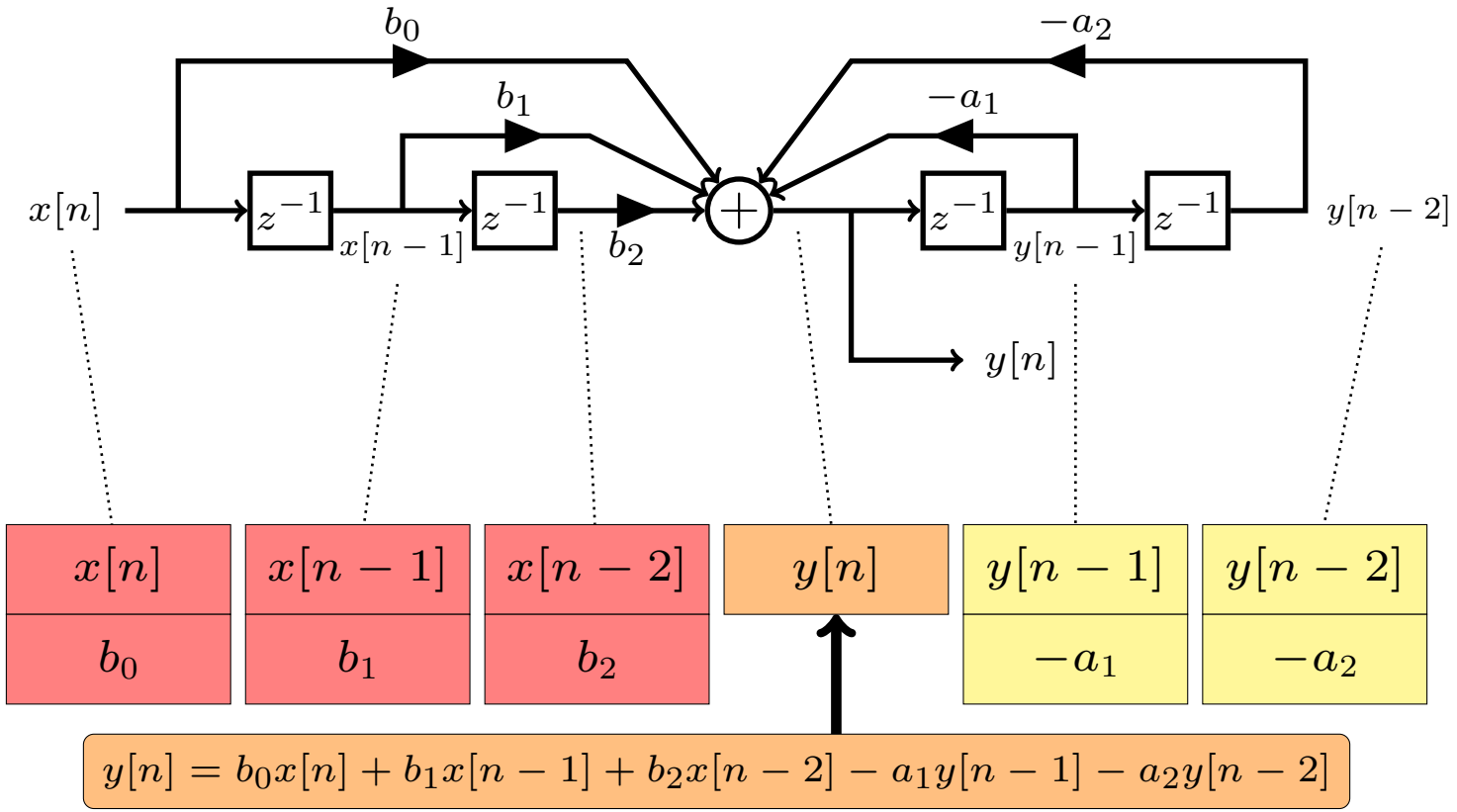
$$a_0 = 1.0000, \quad a_1 = -1.9652, \quad a_2 = 0.9663$$

$$x[n] = \cos(2\pi 0.1n), n = 0, 1, 2, \cdots$$

LTI system magnitude response



block diagram



initialization

n	$x[n]$	$x[n-1]$	$x[n-2]$	$y[n]$	$y[n-1]$	$y[n-2]$
-1	0	0	0	0	0	0

time 0: shift and input new data $x[0]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$y[n]$	$y[n-1]$	$y[n-2]$
-1	0	0	0	0	0	0
0	1.0000	0	0		0	0

time 0: multiply & accumulate

n	$x[n]$	$x[n-1]$	$x[n-2]$	$y[n]$	$y[n-1]$	$y[n-2]$
-1	0	0	0	0	0	0
0	1.0000	0	0	0.0101	0	0
	0.0101	-0.0192	0.0101		-1.9652	0.9663

$$y[0] = 0.0101x[0] - 0.0192x[-1] + 0.0101x[-2] + 1.9652y[-1] - 0.9663y[-2]$$

time 1: shift and input new data $x[1]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$y[n]$	$y[n-1]$	$y[n-2]$
-1	0	0	0	0	0	0
0	1.0000	0	0	0.0101	0	0
1	0.8090	1.000	0		0.0101	0

time 1: multiply & accumulate

n	$x[n]$	$x[n-1]$	$x[n-2]$	$y[n]$	$y[n-1]$	$y[n-2]$
-1	0	0	0	0	0	0
0	1.0000	0	0	0.0101	0	0
1	0.8090	1.000	0	0.0088	0.0101	0
	0.0101	-0.0192	0.0101		-1.9652	0.9663

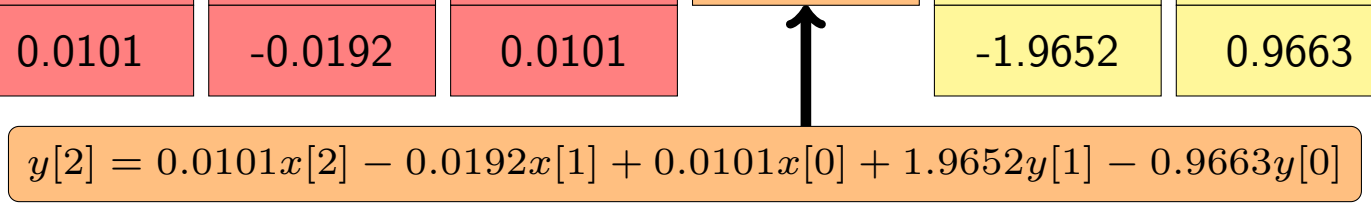
$$y[1] = 0.0101x[1] - 0.0192x[0] + 0.0101x[-1] + 1.9652y[0] - 0.9663y[-1]$$

time 2: shift and input new data $x[2]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$y[n]$	$y[n-1]$	$y[n-2]$
-1	0	0	0	0	0	0
0	1.0000	0	0	0.0101	0	0
1	0.8090	1.000	0	0.0088	0.0101	0
2	0.3090	0.8090	1.0000		0.0088	0.0101

time 2: shift and input new data $x[2]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$y[n]$	$y[n-1]$	$y[n-2]$
-1	0	0	0	0	0	0
0	1.0000	0	0	0.0101	0	0
1	0.8090	1.000	0	0.0088	0.0101	0
2	0.3090	0.8090	1.0000	0.0051	0.0088	0.0101
	0.0101	-0.0192	0.0101		-1.9652	0.9663



$$y[2] = 0.0101x[2] - 0.0192x[1] + 0.0101x[0] + 1.9652y[1] - 0.9663y[0]$$

time 3: shift and input new data $x[3]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$y[n]$	$y[n-1]$	$y[n-2]$
-1	0	0	0	0	0	0
0	1.0000	0	0	0.0101	0	0
1	0.8090	1.000	0	0.0088	0.0101	0
2	0.3090	0.8090	1.0000	0.0051	0.0088	0.0101
3	-0.3090	0.3090	0.8090		0.0051	0.0088

time 3: multiply & accumulate

n	$x[n]$	$x[n-1]$	$x[n-2]$	$y[n]$	$y[n-1]$	$y[n-2]$
-1	0	0	0	0	0	0
0	1.0000	0	0	0.0101	0	0
1	0.8090	1.000	0	0.0088	0.0101	0
2	0.3090	0.8090	1.0000	0.0051	0.0088	0.0101
3	-0.3090	0.3090	0.8090	0.0007	0.0051	0.0088
	0.0101	-0.0192	0.0101		-1.9652	0.9663

$$y[3] = 0.0101x[3] - 0.0192x[2] + 0.0101x[1] + 1.9652y[2] - 0.9663y[1]$$

time 4: shift and input new data $x[4]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$y[n]$	$y[n-1]$	$y[n-2]$
-1	0	0	0	0	0	0
0	1.0000	0	0	0.0101	0	0
1	0.8090	1.000	0	0.0088	0.0101	0
2	0.3090	0.8090	1.0000	0.0051	0.0088	0.0101
3	-0.3090	0.3090	0.8090	0.0007	0.0051	0.0088
4	-0.8090	-0.3090	0.3090		0.0007	0.0051

time 4: multiply & accumulate

n	$x[n]$	$x[n-1]$	$x[n-2]$	$y[n]$	$y[n-1]$	$y[n-2]$
-1	0	0	0	0	0	0
0	1.0000	0	0	0.0101	0	0
1	0.8090	1.000	0	0.0088	0.0101	0
2	0.3090	0.8090	1.0000	0.0051	0.0088	0.0101
3	-0.3090	0.3090	0.8090	0.0007	0.0051	0.0088
4	-0.8090	-0.3090	0.3090	-0.0026	0.0007	0.0051
	0.0101	-0.0192	0.0101		-1.9652	0.9663

$$y[4] = 0.0101x[4] - 0.0192x[3] + 0.0101x[2] + 1.9652y[3] - 0.9663y[2]$$

convolution vs. filtering

The process described on the previous slides produces one output sample for every input sample. It is filtering.

Convolution operates using the impulse response $h[n]$.

Convolution produces a block of outputs given a block of inputs.