ECE 3640 - Discrete-Time Signals and Systems
LTI Systems: FIR Computation

Jake Gunther

Spring 2017

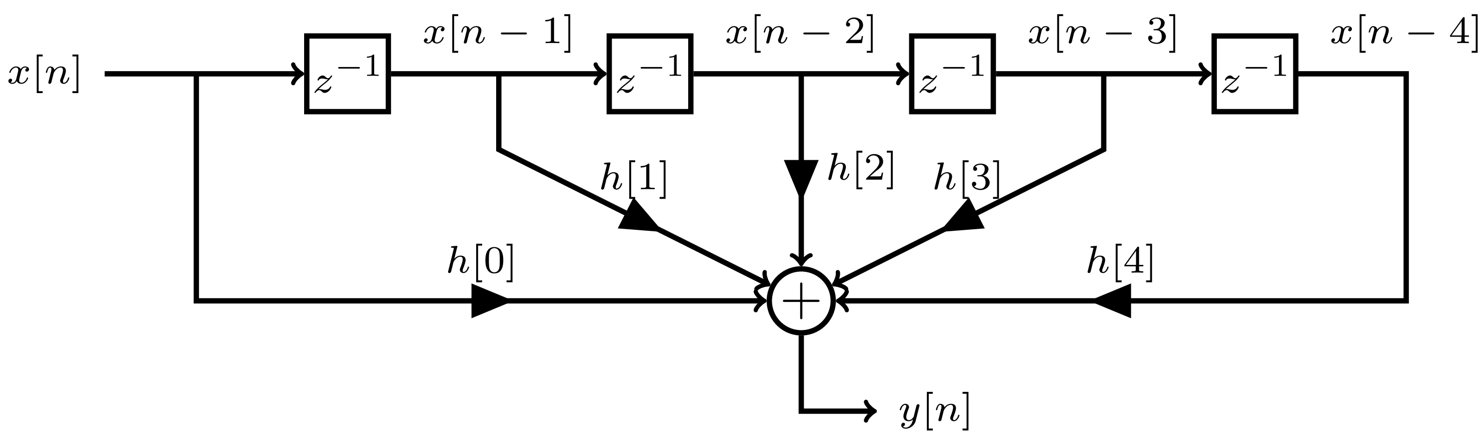
UtahStateUniversity

Department of Electrical & Computer Engineering

main points

- compute the output of DT IIR LTI systems (difference equation)
- compute the output of DT FIR LTI systems (filtering, convolution)

difference equation & block diagram



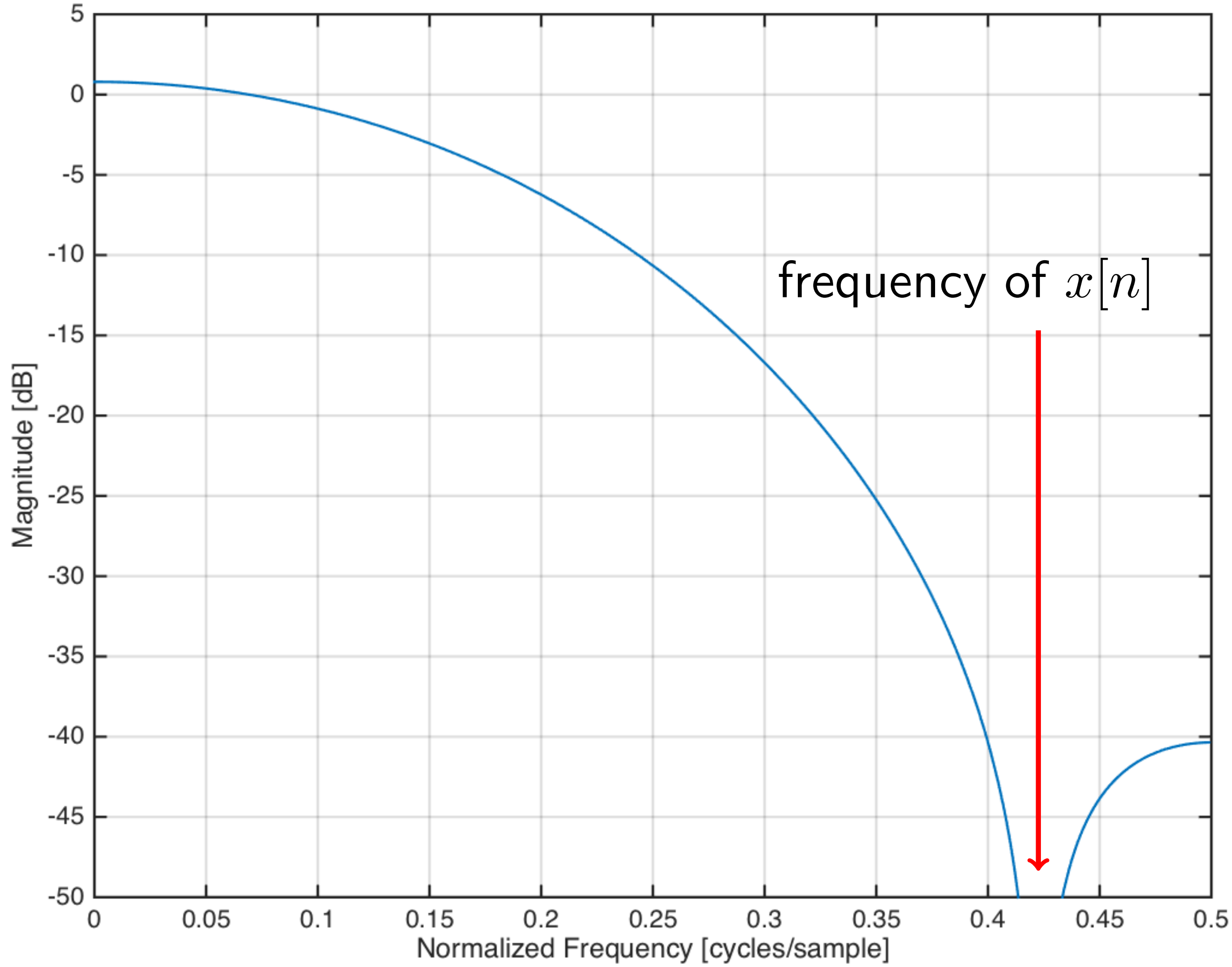
$$y[n] = \sum_{i=0}^{4} h[i]x[n-i] \qquad \text{(convolution sum)}$$

$$h[0] = h[4] = 0.0625, \quad h[1] = h[3] = 0.2764, \quad h[2] = 0.4182$$

$$x[n] = \cos(2\pi 0.422 n), \quad n = 0, 1, 2, \cdots$$

(data)	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$
(coef.)	$h[0]$	$h[1]$	$h[2]$	$h[3]$	$h[4]$

LTI system magnitude response



Because frequency of $x[n]$ coincides with null in magnitude response of filter, the filter output $y[n]$ should be zero.

initialization

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	

time 0: shift & input new data $x[0]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	
0	1.0000	0	0	0	0	

time 0: multiply & accumulate to get $y[0]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	
0	1.0000	0	0	0	0	0.0625
	$h[0]$	$h[1]$	$h[2]$	$h[3]$	$h[4]$	

$$y[n] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + h[3]x[n-3] + h[4]x[n-4]$$

time 1: shift & input new data $x[1]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	
0	1.0000	0	0	0	0	0.0625
1	-0.8823	1.0000	0	0	0	

time 1: multiply & accumulate to get $y[1]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	
0	1.0000	0	0	0	0	0.0625
1	-0.8823	1.0000	0	0	0	0.2212
	$h[0]$	$h[1]$	$h[2]$	$h[3]$	$h[4]$	

$$y[n] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + h[3]x[n-3] + h[4]x[n-4]$$

time 2: shift & input new data $x[2]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	
0	1.0000	0	0	0	0	0.0625
1	-0.8823	1.0000	0	0	0	0.2212
2	0.5569	-0.8823	1.0000	0	0	

time 2: multiply & accumulate to get $y[2]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	
0	1.0000	0	0	0	0	0.0625
1	-0.8823	1.0000	0	0	0	0.2212
2	0.5569	-0.8823	1.0000	0	0	0.2091
	$h[0]$	$h[1]$	$h[2]$	$h[3]$	$h[4]$	

$$y[n] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + h[3]x[n-3] + h[4]x[n-4]$$

time 3: shift & input new data $x[3]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	
0	1.0000	0	0	0	0	0.0625
1	-0.8823	1.0000	0	0	0	0.2212
2	0.5569	-0.8823	1.0000	0	0	0.2091
3	-0.1004	0.5569	-0.8823	1.0000	0	

time 3: multiply & accumulate to get $y[3]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	
0	1.0000	0	0	0	0	0.0625
1	-0.8823	1.0000	0	0	0	0.2212
2	0.5569	-0.8823	1.0000	0	0	0.2091
3	-0.1004	0.5569	-0.8823	1.0000	0	0.0551
	$h[0]$	$h[1]$	$h[2]$	$h[3]$	$h[4]$	

$$y[n] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + h[3]x[n-3] + h[4]x[n-4]$$

time 4: shift & input new data $x[4]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	
0	1.0000	0	0	0	0	0.0625
1	-0.8823	1.0000	0	0	0	0.2212
2	0.5569	-0.8823	1.0000	0	0	0.2091
3	-0.1004	0.5569	-0.8823	1.0000	0	0.0551
4	-0.3798	-0.1004	0.5569	-0.8823	1.0000	

time 4: multiply & accumulate to get $y[4]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	
0	1.0000	0	0	0	0	0.0625
1	-0.8823	1.0000	0	0	0	0.2212
2	0.5569	-0.8823	1.0000	0	0	0.2091
3	-0.1004	0.5569	-0.8823	1.0000	0	0.0551
4	-0.3798	-0.1004	0.5569	-0.8823	1.0000	0.0000
	$h[0]$	$h[1]$	$h[2]$	$h[3]$	$h[4]$	

$$y[n] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + h[3]x[n-3] + h[4]x[n-4]$$

time 5: shift & input new data $x[5]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	
0	1.0000	0	0	0	0	0.0625
1	-0.8823	1.0000	0	0	0	0.2212
2	0.5569	-0.8823	1.0000	0	0	0.2091
3	-0.1004	0.5569	-0.8823	1.0000	0	0.0551
4	-0.3798	-0.1004	0.5569	-0.8823	1.0000	0.0000
5	0.7705	-0.3798	-0.1004	0.5569	-0.8823	

time 5: multiply & accumulate to get $y[5]$

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	
0	1.0000	0	0	0	0	0.0625
1	-0.8823	1.0000	0	0	0	0.2212
2	0.5569	-0.8823	1.0000	0	0	0.2091
3	-0.1004	0.5569	-0.8823	1.0000	0	0.0551
4	-0.3798	-0.1004	0.5569	-0.8823	1.0000	0.0000
5	0.7705	-0.3798	-0.1004	0.5569	-0.8823	0.0000
	$h[0]$	$h[1]$	$h[2]$	$h[3]$	$h[4]$	

$$y[n] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + h[3]x[n-3] + h[4]x[n-4]$$

times 6, 7, 8, 9

n	$x[n]$	$x[n-1]$	$x[n-2]$	$x[n-3]$	$x[n-4]$	$y[n]$
-1	0	0	0	0	0	
0	1.0000	0	0	0	0	0.0625
1	-0.8823	1.0000	0	0	0	0.2212
2	0.5569	-0.8823	1.0000	0	0	0.2091
3	-0.1004	0.5569	-0.8823	1.0000	0	0.0551
4	-0.3798	-0.1004	0.5569	-0.8823	1.0000	0.0000
5	0.7705	-0.3798	-0.1004	0.5569	-0.8823	0.0000
6	-0.9799	0.7705	-0.3798	-0.1004	0.5569	0.0000
7	0.9585	-0.9799	0.7705	-0.3798	-0.1004	0.0000
8	-0.7115	0.9585	-0.9799	0.7705	-0.3798	0.0000
9	0.2970	-0.7115	0.9585	-0.9799	0.7705	0.0000

The output was expected to be zero.
Why are the first few outputs nonzero?
When does the output become zero as expected?

observations

For an FIR filter of length M:

- the first $M-1$ outputs are transients
- the filter does not function as expected until filled with samples